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Research Article

On the Efficiency of Imputation Estimators using Auxiliary Attribute

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Abstract

Surveys such as medical and social science surveys, conducted by human are often associated with problems of non-response or missing observations. Several schemes and estimators have been suggested by authors like Singh and Horn (2000), Singh et al (2014), Prasad (2017) and several others, to estimate the population in such situations. However, the existing schemes and estimators only consider quantitative auxiliary variables not qualitative. In this study, some imputation methods were studied using auxiliary attribute and two new imputation schemes using auxiliary attribute have been suggested. The properties (bias and MSE) of the proposed estimators were derived up to a first order approximation using Taylor series approach. Conditions for which the proposed estimator more efficient than other estimators considered in the study were also established. Numerical illustration was conducted and the results revealed that the proposed estimator is more efficient.

Keywords: Imputation, Non-response, Estimator, Population Mean, Attribute.

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Introduction

Studies in sample survey have established the fact that auxiliary characters play a significant role in the improvement of precision of estimators of population characteristics especially when the study and auxiliary variables are strongly associated. This concept gave birth to ratio, product, dual-to-ratio and regression estimators. Also, several authors have employed the concept of auxiliary variables in the development and improvement of imputation schemes to obtain information for non-respondents in the surveys. These authors, including Singh and Horn (2000), Singh and Deo (2003), Wang and Wang (2006), Toutenburg et al. (2008), Kadilar and Cingi (2008), Singh (2009), Diana and Perri (2010), Al-Omari et al. (2013), Singh et al. (2014), Gira (2015), Singh et al. (2016), Bhushan and Pandey (2016), Prasad (2017). Situations arise when the auxiliary characters are qualitative in nature e.g. gender, marital status, family history on a disease, patient status with respect to disease, and of which the imputation schemes proposed by aforementioned authors will be impracticable. In the present study, we consider generalized imputation schemes when the auxiliary character is qualitative.

Existing Imputation Schemes using Auxiliary Attribute

Let H denotes the set of r units response and H^c denotes the set of $n-r$ units non-response or missing out of n units sampled without replacement from the N units population. For each unit $i \in \mathrm{H}$, the value of $y_{.i}$ is observed. However, for unit $i \in \mathrm{H}^c$, $y_{.i}$ is missing but calculated using different methods of imputation.

The mean method of imputation, values found missing are to be replaced by the mean of the rest of observed values. The study variable thereafter, takes the form given as,

$$y_{.i} = \begin{cases} y_i & \text{if} \quad i \in \mathrm{H} \\ \bar{y}_r & \text{if} \quad i \in \mathrm{H}^c \end{cases} \tag{2.1}$$

Under the method of imputation, sample mean denoted by $\hat{t}_0$ can be derived as

$$\hat{t}_0 = \frac{1}{r}\sum_{i \in R} y_i \tag{2.2}$$

The variance of $\hat{t}_0$ is given by (2.3).

$$Var(\hat{t}_0) = \theta_r \bar{Y}^2 C_Y^2 \tag{2.3}$$

where $\theta_r = \frac{1}{r} - \frac{1}{N}$, $C_Y = S_Y^2 / \bar{Y}, S_Y^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^2$, $\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$

Ratio method of imputation when auxiliary character is attributed is defined as

$$y_{.i} = \begin{cases} y_i & \text{if} \quad i \in \text{H} \\ \hat{\beta}^* \phi_1 & \text{if} \quad i \in \text{H}^c \end{cases}$$

(2.4)

where $\hat{\beta}^* = \sum_{i=1}^{r} y_i / \sum_{i=1}^{r} \phi_i = \bar{y}_r / p_r$, $p_r = \frac{1}{r}\sum_{i \in R} \phi_i$.

The estimator of population mean denoted by $\hat{t}_1$ of (2.4) is given as

$$\hat{t}_1 = \bar{y}_r \frac{p_n}{p_r} \tag{2.5}$$

where $p_n = \frac{1}{n}\sum_{i \in S} \phi_i$

The MSE of $\hat{t}_1$ up $\text{O}\left(n^{-1}\right)$ is given as:

$$MSE(\hat{t}_1) = \bar{Y}^2 \left(\theta_r C_Y^2 + \theta_n \left(C_\phi^2 - 2\rho_{y\phi} C_Y C_\phi\right)\right) \tag{2.6}$$

where

$$\rho_{Y\phi} = \frac{S_{Y\phi}}{S_Y S_\phi}, C_\phi = S_\phi^2 / P,\ S_\phi^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(\phi_i - P\right)^2, P = \frac{1}{N}\sum_{i=1}^{N}\phi_i,\ \theta_n = \frac{1}{r} - \frac{1}{n},$$

$$S_{Y\phi} = \frac{1}{N-1}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)\left(\phi_i - P\right)$$

Imputation scheme proposed by Singh and Horn (2000) when auxiliary character is qualitative in nature is defined as

$$y_{.i} = \begin{cases} \lambda \frac{n}{r} y_i + (1-\lambda)\hat{\beta}^* \phi_i & \text{if} \quad i \in \text{H} \\ (1-\lambda)\hat{\beta}^* \phi_i & \text{if} \quad i \in \text{H}^c \end{cases} \tag{2.7}$$

Under this scheme, the estimator of population mean denoted by $\hat{t}_2$ and its MSE are given as

$$\hat{t}_2 = \bar{y}_r\left(\lambda + (1-\lambda)\frac{p_n}{p_r}\right) \tag{2.8}$$

$$MSE\left(\hat{t}_2\right)_{\min} = \bar{Y}^2 C_Y^2\left(\theta_r - \theta_n \rho_{Y\phi}^2\right) \tag{2.9}$$

Imputation scheme proposed by Singh and Deo (2003) when auxiliary character is qualitative in nature is defined as

$$y_{.i} = \begin{cases} y_i & \text{if} \quad i \in \mathrm{H} \\ \dfrac{\bar{y}_r}{n-r}\left(n\left(\dfrac{p_n}{p_r}\right)^{\alpha} - r\right) & \text{if} \quad i \in \mathrm{H}^c \end{cases} \tag{2.10}$$

Under this scheme, the estimator of population mean denoted by $\hat{t}_3$ and its MSE are given as

$$\hat{t}_3 = \bar{y}_r\left(\frac{p_n}{p_r}\right)^{\alpha} \tag{2.10}$$

$$MSE(\hat{t}_3)_{\min} = \bar{Y}^2 C_Y^2\left(\theta_r - \theta_n \rho_{Y\phi}^2\right) \tag{2.11}$$

Imputation scheme proposed by Ahmed et al. (2006) when auxiliary character is qualitative in nature is defined as

$$y_{.i} = \begin{cases} y_i & \text{if} \quad i \in \mathrm{H} \\ \dfrac{\bar{y}_r}{n-r}\left(n\left(\dfrac{P}{p_r}\right)^{\beta} - r\right) & \text{if} \quad i \in \mathrm{H}^c \end{cases} \tag{2.12}$$

Under this scheme, the estimator of population mean denoted by $\hat{t}_4$ and its MSE are given as

$$\hat{t}_4 = \bar{y}_r\left(\frac{P}{p_r}\right)^{\beta} \tag{2.13}$$

$$MSE(\hat{t}_4) = \bar{Y}^2 C_Y^2\left(\theta_r - \theta_n \rho_{Y\phi}^2\right) \tag{2.14}$$

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Imputation scheme proposed by Singh (2009) when auxiliary character is qualitative in nature is defined as

$$y_{.i} = \begin{cases} y_i & \text{if} \quad i \in \mathrm{H} \\ \bar{y}_r \left(\dfrac{(n-r)\,p_n + \alpha r\,(p_n - p_r)}{\alpha\, p_r + (1-\alpha)\, p_n} \right) \dfrac{\phi_i}{\sum_{i \in R^c} \phi_i} & \text{if} \quad i \in \mathrm{H}^c \end{cases} \tag{2.15}$$

Under this scheme, the estimator of population mean denoted by $\hat{t}_5$ and its MSE are given as

$$\hat{t}_5 = \frac{\bar{y}_r p_n}{\alpha p_r + (1-\alpha)\, p_n} \tag{2.16}$$

$$MSE(\hat{t}_5)_{\min} = \bar{Y}^2 C_Y^2 \left(\theta_r - \theta_n \rho_{Y\phi}^2\right) \tag{2.17}$$

Singh et al. (2014) proposed Exponential-Type Compromised Imputation method as

$$y_{.i} = \begin{cases} k\dfrac{n}{r} y_i + (1-k)\,\bar{y}_r \exp\left(\dfrac{P - p_r}{P + p_r}\right) & \text{if} \quad i \in \mathrm{H} \\ (1-k)\,\bar{y}_r \exp\left(\dfrac{P - p_r}{P + p_r}\right) & \text{if} \quad i \in \mathrm{H}^c \end{cases} \tag{2.18}$$

Under this scheme, the estimator of population mean denoted by $\hat{t}_6$ and its MSE are given as

$$\hat{t}_6 = \kappa \bar{y}_r + (1-\kappa)\,\bar{y}_r \exp\left(\frac{P - p_r}{P + p_r}\right) \tag{2.19}$$

$$MSE\left(\hat{t}_6\right)_{\min} = \theta_r \bar{Y}^2 C_Y^2 \left(1 - \rho_{Y\phi}^2\right) \tag{2.20}$$

Singh and Gogoi (2017) proposed Dual-to-Ratio Exponential-Type Compromised Imputation method given as

$$y_{.i} = \begin{cases} w\dfrac{n}{r} y_i + (1-w)\,\bar{y}_r \exp\left(\dfrac{p^* - P}{p^* + P}\right) & \text{if} \quad i \in \mathrm{H} \\ (1-w)\,\bar{y}_r \exp\left(\dfrac{p^* - P}{p^* + P}\right) & \text{if} \quad i \in \mathrm{H}^c \end{cases} \tag{2.21}$$

where $p^* = (NP - np_r)/(N-n)$.

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Under this scheme, the estimator of population mean denoted by $\hat{t}_7$ and its MSE are given as

$$\hat{t}_7 = w\bar{y}_r + (1-w)\,\bar{y}_r \exp\left(\frac{p^* - P}{p^* + P}\right) \tag{2.22}$$

$$MSE\left(\hat{t}_7\right)_{\min} = \theta_r \bar{Y}^2 C_Y^2 \left(1-\rho_{Y\phi}^2\right) \tag{2.23}$$

However, critically observing the above schemes, it is observed that as the values unknown functions in the schemes approaches unity, the components of the schemes associated with auxiliary characters converges to zero which in turn diminish the efficiency of the schemes. To overcome this shortcoming, new imputation schemes were proposed in this study.

Proposed New Imputation Schemes

Having study the imputation scheme of Singh et al. (2014), we proposed new imputation scheme as;

$$y_{.i} = \begin{cases} y_i & \text{if } \; i \in \mathrm{H} \\ \dfrac{\bar{y}_r}{n-r}\left(n\left(\lambda_1 \dfrac{P}{p_r} + \lambda_2 \dfrac{p_r}{P}\right)\exp\left(\dfrac{p_r - P}{p_r + P}\right) - r\right) & \text{if } \; i \in \mathrm{H}^c \end{cases} \tag{3.1}$$

where $\lambda_1 \neq 0$ and $\lambda_2 \neq 0$ are unknown functions of study variable and auxiliary attribute.

The point estimatorsof finite population mean under this scheme denoted by t1 is given by

$$\hat{t}_i^{(*)} = \bar{y}_r\left(\lambda_1 \frac{P}{p_r} + \lambda_2 \frac{p_r}{P}\right)\exp\left(\frac{p_r - P}{p_r + P}\right) \tag{3.2}$$

Also, having study the imputation scheme of Singh and Gogoi (2017), we proposed new imputation scheme as;

$$y_{.i} = \begin{cases} y_i & \text{if } \; i \in \mathrm{H} \\ \dfrac{\bar{y}_r}{n-r}\left(n\left(\pi_1 \dfrac{P}{p^*} + \pi_2 \dfrac{p^*}{P}\right)\exp\left(\dfrac{P - p^*}{P + p^*}\right) - r\right) & \text{if } \; i \in \mathrm{H}^c \end{cases} \tag{3.3}$$

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where $\pi_1 \neq 0$ and $\pi_2 \neq 0$ are unknown functions of study variable and auxiliary attribute.

The point estimators of finite population mean under this scheme denoted by t2 is given by

$$\hat{t}_i^{(*)} = \bar{y}_r\left(\pi_1 \frac{P}{p^*} + \pi_2 \frac{p^*}{P}\right)\exp\left(\frac{P - p^*}{P + p^*}\right) \tag{3.4}$$

Properties of the Estimators of the New Imputation Schemes

In this section, the bias and MSE of the estimators suggested in this paper are derived and discussed.

Let $\bar{y}_r = \bar{Y}(1+e_0)$, $p_r = P(1+e_1)$ such that $|e_i| \approx 0, i = 1,2.$ with expectation defined as

$$\left.\begin{aligned} &E(e_0) = E(e_1) = 0, E(e_0^2) = \theta_r C_Y^2 \\ &E(e_1^2) = \theta_r C_\phi^2, E(e_0 e_1) = \theta_r \rho_{Y\phi} C_Y C_\phi \end{aligned}\right\} \tag{4.1}$$

Expressing (3.1) and (3.2) in terms of e_i up to second degree approximation, we have

$$t_1^* = \bar{Y}\left(\Lambda_1 + \Lambda_2 - (\Lambda_1 - \Lambda_2)e_1 + \Lambda_1 e_1^2 + (\Lambda_1 + \Lambda_2)e_0 - (\Lambda_1 - \Lambda_2)e_0 e_1\right)\left(1 - \frac{e_1}{2} + \frac{3e_1^2}{8}\right) \tag{4.2}$$

$$\begin{aligned} t_2^* = \bar{Y}\Bigg(&\mathrm{H}_1 + \mathrm{H}_2 - \frac{n(\mathrm{H}_1 - \mathrm{H}_2)e_1}{N-n} + \frac{n^2 \mathrm{H}_1 e_1^2}{(N-n)^2} + \frac{n(\mathrm{H}_1 + \mathrm{H}_2)e_0}{N-n} \\ &- \frac{n(\mathrm{H}_1 - \mathrm{H}_2)e_0 e_1}{N-n}\Bigg)\left(1 - \frac{ne_1}{2(N-n)} + \frac{3n^2 e_1^2}{8(N-n)^2}\right) \end{aligned} \tag{4.3}$$

Subtract $\bar{Y}$ from (4.2) and (4.3) simplify the results up to second degree approximation, we have

$$t_1^* - \bar{Y} = \bar{Y}\left(\Lambda_1\left(1 - \frac{3e_1}{2} + e_0 + \frac{15e_1^2}{8} - \frac{9e_0 e_1}{8}\right) + \Lambda_2\left(1 + \frac{e_1}{2} + e_0 - \frac{e_1^2}{8} + \frac{7e_0 e_1}{8}\right) - 1\right) \tag{4.4}$$

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$$t_2^* - \bar{Y} = \bar{Y}\left(\mathrm{H}_1\left(1+e_0+\frac{M}{2}\left(e_1+\frac{3Me_1^2}{4}+e_0e_1\right)\right)+\mathrm{H}_2\left(1+e_0-\frac{3M}{2}\left(e_1-\frac{Me_1^2}{4}+e_0e_1\right)\right)-1\right) \quad (4.5)$$

where $M = n/(N-n)$

Take expectation of (4.4) and (4.5) and apply the results of (4.1), we obtain the biases of t_1^* and t_2^* as

$$Bias\left(t_1^*\right) \approx \bar{Y}\left(\Lambda_1\left(1+\frac{\theta_r}{8}\left(3C_p^2-4\rho C_Y C_p\right)\right)+\Lambda_2\left(1+\frac{\theta_r}{8}\left(3C_p^2-12\rho C_Y C_p\right)\right)-1\right) \quad (4.6)$$

$$Bias\left(t_2^*\right) \approx \bar{Y}\left(\mathrm{H}_1\left(1+\frac{\theta_r M}{8}\left(3MC_p^2+4\rho C_Y C_p\right)\right)+\mathrm{H}_2\left(1+\frac{3\theta_r M}{8}\left(MC_p^2-4\rho C_Y C_p\right)\right)-1\right) \quad (4.7)$$

Square (4.4) and (4.5), take expectation and apply the results of (4.1), we obtain the MSEs of t_1^* and t_2^* as

$$MSE\left(t_1^*\right) = \bar{Y}^2\left(1+\Lambda_1^2\Psi_1+\Lambda_2^2\Psi_2-2\Lambda_1\Psi_3-2\Lambda_2\Psi_4+2\Lambda_2\Lambda_2\Psi_5\right) \quad (4.8)$$

$$MSE\left(t_2^*\right) = \bar{Y}^2\left(1+\mathrm{H}_1^2\Theta_1+\mathrm{H}_2^2\Theta_2-2\mathrm{H}_1\Theta_3-2\mathrm{H}_2\Theta_4+2\mathrm{H}_1\mathrm{H}_2\Theta_5\right) \quad (4.9)$$

where

$$\Psi_1 = 1+\theta_r\left(C_Y^2+\frac{1}{2}C_P^2-2\rho C_Y C_P\right), \Psi_2 = 1+\theta_r\left(C_Y^2+3C_X^2+6\rho C_Y C_P\right), \Psi_3 = 1+\frac{\theta_r}{8}\left(3C_P^2-4\rho C_Y C_P\right)$$

$$\Psi_4 = 1+\frac{\theta_r}{8}\left(3C_P^2-12\rho C_Y C_P\right),\ \Psi_5 = 1+\theta_r\left(C_Y^2+2\rho C_Y C_P\right)$$

$$\Theta_1 = 1+\theta_r\left(C_Y^2+M^2C_P^2+2M\rho C_Y C_P\right), \Theta_2 = 1+\theta_r\left(C_Y^2+3M^2C_P^2-6M\rho C_Y C_P\right),$$

$$\Theta_3 = 1+\frac{3\theta_r}{8}\left(M^2C_P^2+\frac{4}{3}\rho C_Y C_P\right), \Theta_4 = 1+\frac{3\theta_r M}{8}\left(MC_P^2-4\rho C_Y C_P\right),\ \Theta_5 = 1+\theta_r\left(C_Y^2-2M\rho C_Y C_P\right)$$

Differentiate (4.8) partially with respect to Λ_1 and Λ_2, equate the results to zeros, we system of linear equation as;

$$\left.\begin{aligned} \Lambda_1\Psi_1+\Lambda_2\Psi_5 &= \Psi_3 \\ \Lambda_1\Psi_5+\Lambda_2\Psi_2 &= \Psi_4 \end{aligned}\right\} \quad (4.10)$$

Solve (4.10), we obtain

$$\Lambda_1 = \frac{\Psi_2\Psi_3 - \Psi_4\Psi_5}{\Psi_1\Psi_2 - \Psi_5^2},\ \Lambda_2 = \frac{\Psi_1\Psi_4 - \Psi_3\Psi_5}{\Psi_1\Psi_2 - \Psi_5^2} \tag{4.11}$$

Substitute (4.11) in (4.8), we obtain minimum MSE of t_1^* as

$$MSE\left(t_1^*\right)_{\min} = \bar{Y}^2\left(1 - \frac{\Psi_2\Psi_3^2 + \Psi_1\Psi_4^2 - 2\Psi_3\Psi_4\Psi_5}{\Psi_1\Psi_2 - \Psi_5^2}\right) \tag{4.12}$$

Also, differentiate (4.9) partially with respect to H_1 and H_2, equate the results to zeros, we system of linear equation as;

$$\left.\begin{aligned} \mathrm{H}_1\Theta_1 + \mathrm{H}_2\Theta_5 &= \Theta_3 \\ \mathrm{H}_1\Theta_5 + \mathrm{H}_2\Theta_2 &= \Theta_4 \end{aligned}\right\} \tag{4.13}$$

Solve (4.12), we obtain

$$\mathrm{H}_1 = \frac{\Theta_2\Theta_3 - \Theta_4\Theta_5}{\Theta_1\Theta_2 - \Theta_5^2},\ \mathrm{H}_2 = \frac{\Theta_1\Theta_4 - \Theta_3\Theta_5}{\Theta_1\Theta_2 - \Theta_5^2} \tag{4.14}$$

Substitute (4.14) in (4.9), we obtain minimum MSE of t_2^* as

$$MSE\left(t_2^*\right)_{\min} = \bar{Y}^2\left(1 - \frac{\Theta_2\Theta_3^2 + \Theta_1\Theta_4^2 - 2\Theta_3\Theta_4\Theta_5}{\Theta_1\Theta_2 - \Theta_5^2}\right) \tag{4.15}$$

Efficiency Comparison

In this section, conditions for the efficiency of the new estimators over some existing related estimators were established.

Estimators $\hat{t}_i^*,\ i = 1,2$ is more efficient than $\hat{t}_0$ if;

$$MSE\left(\hat{t}_0\right) - MSE\left(\hat{t}_i^*\right) > 0 \quad \Rightarrow \quad \theta_r C_Y^2 + \mathrm{K}_{i1}/\mathrm{K}_{i2} - 1 > 0 \tag{5.1}$$

Estimators $\hat{t}_i^*,\ i = 1,2$ is more efficient than $\hat{t}_1$ if;

$$MSE\left(\hat{t}_1\right) - MSE\left(\hat{t}_i^*\right) > 0 \ \Rightarrow \theta_r C_Y^2 + \theta_n\left(C_\phi^2 - 2\rho_{Y\phi}C_Y C_\phi\right) + \mathrm{K}_{i1}/\mathrm{K}_{i2} - 1 > 0 \tag{5.2}$$

Estimators $\hat{t}_i^*,\ i = 1,2$ is more efficient than $\hat{t}_j,\ j = 2,3,4,5,6,7$ if;

$$MSE\left(\hat{t}_j\right) - MSE\left(\hat{t}_i^*\right) > 0 \quad \Rightarrow \left(\theta_r - \theta_n\rho_{Y\phi}^2\right)C_Y^2 + \mathrm{K}_{i1}/\mathrm{K}_{i2} - 1 > 0 \tag{5.3}$$

where $\begin{aligned} &\mathrm{K}_{11} = \Psi_2\Psi_3^2 + \Psi_1\Psi_4^2 - 2\Psi_3\Psi_4\Psi_5,\ \mathrm{K}_{12} = \Psi_1\Psi_2 - \Psi_5^2, \\ &\mathrm{K}_{21} = \Theta_2\Theta_3^2 + \Theta_1\Theta_4^2 - 2\Theta_3\Theta_4\Theta_5, \quad \mathrm{K}_{22} = \Theta_1\Theta_2 - \Theta_5^2 \end{aligned}$

Proposed estimators are more efficient than other estimators considered in the study if the inequalities (5.1)-(5.3) are satisfied.

Numerical illustration

For the empirical justification of the results, we consider five sets of real data. The performance of the proposed estimator is justified by comparing its MSE to those of some existing estimators considered in the study.

Population I: Source (Sukhatme and Sukhatme, 1970)

$Y =$ The number of villages in the circle,

$$\phi = \begin{cases} 1, & if\ Y > 5 \\ 0, & if\ Y \leq 5 \end{cases}$$

Population II: Source (Zaman et al., 2014)

$Y =$ The number of teachers,

$$\phi = \begin{cases} 1, & if\ Y > 5 \\ 0, & if\ Y \leq 5 \end{cases}$$

Table 1: Descriptive Statistics of the Populations

Population I: $N = 89, n = 20, \bar{Y} = 3.3596, \rho_{Y\phi} = 0.766, C_Y = 0.6008, C_\phi = 2.6779$
Population II: $N = 111, n = 30, \bar{Y} = 29.279, \rho_{Y\phi} = 0.797, C_Y = 0.872, C_\phi = 2.758$

Table 2: MSE Values of Some and Proposed Estimators using Population I Data

Number of respondents r	Estimators				
	$\hat{t}_0$	$\hat{t}_1$	$\hat{t}_j, j = 2,3,4,6,7$	$\hat{t}_1^*$	$\hat{t}_2^*$
18	0.1805639	0.4756753	0.1632262	0.288260	0.0740248
14	0.2452326	1.383519	0.1783587	0.3333063	0.1002481
10	0.3616364	3.017639	0.2055971	0.3604635	0.1470652
6	0.633246	6.830585	0.2691536	0.2374562	0.254374

Table 3: MSE Values of Some and Proposed Estimators using Population II Data

Number of respondents r	Estimators				
	$\hat{t}_0$	$\hat{t}_1$	$\hat{t}_j, j=2,3,4,6,7$	$\hat{t}_1^*$	$\hat{t}_2^*$
25	20.20137	41.79199	16.74224	11.46155	7.371982
20	26.71983	80.69638	18.07201	13.3500	9.73795
15	37.58395	145.5370	20.28829	14.94201	13.66706
10	59.31217	275.2183	24.72084	13.41735	21.47085

Tables 2 and 3 show the numerical results of MSE for estimators $\hat{t}_0, \hat{t}_1, \hat{t}_j, j=2,3,...,7, \hat{t}_1^*, \hat{t}_2^*$ using population sets I and II respectively. Of all the estimators considered in the study, the proposed estimators have minimum MSEs for all the two population sets except for data set I where estimators $\hat{t}_j, j=2,3,...,7$ outperformed proposed estimator $\hat{t}_1^*$ when $r=10,14,18$. This implies that the proposed estimators demonstrate high level of efficiency over others and can produce better estimate of population mean in the presence of non-response on the average.

Conclusion

From the results of the numerical illustration in section 4, it was observed that the proposed estimator $\hat{t}_2^*$ is more efficient than other estimators considered in the study and therefore, it is recommended for use for estimating population mean when the study variable is associated with an attribute in the presence of non-response.

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