As per UGC guidelines an electronic bar code is provided to seure your paper

International Journal for Modern Trends in Science and Technology, 8(06): 569-573, 2022

ISSN: 2455-3778 online
DOI: https://doi.org/10.46501/IJMTST0806097
Available online at: http://www.ijmtst.com/vol8issue06.html

Future and Current General Integral Transform for Solving Integral Equations

Manish Gaur, Ashwin Singh Chouhan

Jai Narain Vyas University (New Campus), Jodhpur (Raj.)India 342003
Email Id: ashwinsingh26061992@gmail.com

To Cite this Article
Manish Gaur and Ashwin Singh Chouhan. Future and Current General Integral Transform for Solving Integral Equations. International Journal for Modern Trends in Science and Technology 2022, 8(06), pp. 569-573. https://doi.org/10.46501/IJMTST0806097

Article Info
Received: 25 May 2022; Accepted: 24 June 2022; Published: 26 June 2022.

ABSTRACT

Background: The subject of Integral equations is one of the most useful mathematical tools in both pure and applied mathematics. It has enormous applications in many physical problems. Integral transforms are important to solve real problems.

Method and material: We conducted this research paper by observing the different types of reviews, as well as conducting and evaluating literature review papers.

Result: A new general integral transform that covers all classes of integral transforms in the class of Laplace transforms. We investigated the application of this new transformation to solving ODEs with constant and variable coefficients.

Conclusions: In this paper, we have discussed the classifications of integral equations such as Fredholm Integral Equation, Volterra Integral Equation and many others along with the kinds of particular equation.

KEYWORDS: Integral Equation, Classification, Kernels, Relation of Differential Integral Equations etc

INTRODUCTION

The subject of Integral equations is one of the most useful mathematical tools in both pure and applied mathematics. It has enormous applications in many physical problems. Many initial and boundary value problems associated with ODE (ordinary differential equations) and PDE (partial differential equations) can be transformed into problems of solving some approximate integral equations. Integral equations were first encountered in the theory of Fourier Integral. In 1826, another integral equation was obtained by Abel. Actual development of the theory of integral equations began with the works of the Italian Mathematician V.Volterra (1896) and the Swedish Mathematician I.Fredholm (1900).

Integral equation

An integral equation is an equation in which the unknown function μ (ϰ) to be determined appears under the integral sign. A typical form of an integral equation in μ (ϰ) () is of the form

$$g(s) = f(s) + \int_{\alpha(s)}^{\beta(s)} k(s,t)\, g(t)dt$$

Where

k (ϰ, t) is called the kernel of the integral equation and α (ϰ) and β (ϰ) are the limit of integration.

Classification of Linear Integral Equations

The most frequently used linear integral equations fall under two main classes namely Fredholm and Volterra integral equations. However, in this text we will distinguish four more related types of linear integral equations in addition to the two main classes. In the following is the list of the Fredholm and Volterra integral equations, and the four more related types:

1. Fredholm Integral Equations
2. Volterra Integral Equations
3. Integro-differential Equations
4. Singular Integral Equations
5. Volterra-Fredholm Integral Equations
6. Volterra-FredholmIntegro-differential Equations

Fredholm Linear Integral Equation

The standard form of Fredholm linear integral equations, where the limits of integration and are constants, are given by the form

$$\phi(s)g(s) = f(s) + \lambda \int_a^b k(s,t)\, g(t)dt \qquad a \le s, t \le b$$

Where the kernel of the integral equation k (ϰ, t) and the function f(s) are given in advance, and λ is a parameter. The equation is called linear because the unknown function g(s) under the integral sign occurs linearly, i.e. the power of g(s)is one.

The value of $\phi(x)$ will give the following kinds of Fredholm linear integral equation

1. When $\phi(x) = 0$, equation

$$f(s) + \lambda \int_a^b k(s,t)\, g(t)dt$$

2. When & $\phi(s) = 1$, equation

$$g(s) + \lambda \int_a^b k(s,t)\, g(t)dt$$

Volterra Linear Integral Equation

The standard form of Volterra linear integral equations, where the limits of integration are functions of rather than constants are of the form

$$\phi(s)g(s) = f(s) + \lambda \int_a^s k(s,t)\, g(t)dt$$

Where the unknown function g(s) under the integral sign occurs linearly as stated before. It is worth noting that can be viewed as a special case of the Fredholm integral equation when the kernel k (ϰ, t) vanishes for t>s,s, is in the range of integration ,[a,b] . As in Fredholm equations, Volterra integral equations fall under two kinds,

1. When $\phi(x) = 0$, equation

$$f(s) + \lambda \int_a^s k(s,t)\, g(t)dt$$

2. When & $\phi(s) = 1$, equation

$$f(s) + \lambda \int_a^s k(s,t)\, g(t)dt$$

REMARKS

1. The structure of Fredholm and Volterra equations: The unknown function g(s) appears linearly only under the integral sign in linear Fredholm and Volterra integral equations of the First Kind. However, the unknown function $g(s)$appears linearly inside as well as outside the integral sign in second kind of both linear Fredholm and Volterra integral equations.

2. The Limits of Integration: In Fredholm integral equations, the integral is taken over a finite interval with fixed limits of integration. However, in Volterra integral equation, at least one limit of the range of integration is a variable, and the upper limit is the most commonly used with a variable limit.

3. The Linearity property: As indicated earlier, the unknown function $g(s)$ in linear Fredholm and Volterra integral equations (1.10) and (1.13) occurs to the first power wherever it exists. However, nonlinear Fredholm and Volterra integral equations arise if $g(s)$ is replaced by a nonlinear function - F ($g(s)$), such as $g2(s), eg(s)$ and so on.

4. The Homogeneity property: On setting $f(s) = 0$ in Fredholm or Volterra integral equation of the second kind given), the resulting equation is called a homogeneous integral equation, otherwise it is called nonhomogeneous integral equation.

Integro-Differential Equations

In this type of equations, the unknown function $g(s)$ occurs in one side as an ordinary derivative, and appears on the other side under the integral sign. Further, we point out that an Integro-differential equation can be easily observed as an intermediate stage when we convert a differential equation to an integral equation

The following are examples of Integro-differential equations:

1. $g''(s) = -s + \int_0^s (s-t)\, g(t)dt$, $g(0) = 0, g'(0)$
2. $g''(s) = \sin s + \int_0^s (s-t)\, g(t)dt$, $g(0) = 1$
3. $g''(s) = 1 - \frac{1}{3}s + \int_0^1 st\; g(t)dt$, $g(0) = 1$

Singular Integral Equations

The integral equation of first kind

$$f(s) + \int_{\alpha(s)}^{\beta(s)} k(s,t)\, g(t)dt$$

Or the integral equation of second kind

$g(s) = f(s) + \int_{\alpha(s)}^{\beta(s)} k(s,t)\, g(t)dt$

Is called singular if the lower limit, the upper limit or both limits of integration are infinite. In addition, the equation is also called singular integral equation if the kernel

$k(s,t)$ becomes infinite at one or more points in the domain of integration.

Volterra-Fredholm integral equations

The Volterra-Fredholm integral equation, which is a combination of disjoint Volterra and Fredholm integrals, appears in one integral equation. The Volterra-Fredholm integral equations arise from the modeling of the spatiotemporal development of an epidemic, from boundary value problems and from many physical and chemical applications. The standard form of the Volterra-Fredholm integral equation reads

$g(s) = f(s) + \lambda \int_0^s k1(s,t)\, g(t)dt + \int_a^b k2(s,t)\, g(t)dt$

Where $k1(s,t)$ and $k2(s,t)$ are the kernels of the equation.

Volterra-FredholmIntegro Differential Equations

The Volterra-Fredholm Differential Equation, which is a combination of disjoint Volterra and Fredholm integrals and Differential operator, may appear in one integral equation. The Volterra-FradholmIntegro-Differential equations arise from many physical and chemical applications similar to the Volterra-Fredholm equations. The standard form of Volterra-FradholmIntegro-Differential equation reads

$gn(s) = f(s) + \int_0^s k1(s,t)\, g(t)dt + \int_a^b k2(s,t)\, g(t)dt$

Where $k1(s,t)$ and $k2(s,t)$ are the kernels of the equation, and n is the order of the ordinary derivative of $g(s)$. Notice that because this kind of equations contains ordinary derivatives, then initial conditions should be prescribed depending on the order of the derivative involved.

Relations between differential and integral equations

To convert the Differential Equations to Integral equations, the following results are necessary:

Leibnitz Rule of Differentiating Under the Integral Sign

If $F(x,t)$ and $\frac{\partial F(x,t)}{\partial x}$(,) are continuous functions of x and t in the domain $\alpha \le x \le, \beta \le t_0 \le t \le t_1$,

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(x,t)dt$$

$$= \int_{a(x)}^{b(x)} \frac{\partial F(x,t)}{\partial x} dt + \frac{db(x)}{dx} f(x, b(x)) - \frac{da(x)}{dx} f(x, a(x))$$

Provided the limits of integration $a(x)$ and $b(x)$ are defined functions having continuous derivatives for $a \le x \le \beta$. This rule may be used to convert integral equations to equivalent ordinary differential equations. In particular, we have

(i) For Volterra Integral Equation:

$$\frac{d}{dx}\left[\int_a^x k(x,t)\, \mu(t) dt = \int_a^x \frac{\partial k}{\partial x} u(t)dt + k(x,x)u(x)\right.$$

(ii) For Fredholm Integral Equation:

$$\frac{d}{dx}\left[\int_a^x k(x,t)\, \mu(t) dt = \int_a^x \frac{\partial k}{\partial x} u(t)dt\right.$$

Here $\mu(t)$ is independent of x and hence on taking partial derivatives with respect to x, $\mu(t)$ is treated as constant.

Cauchy's Formula for Repeated Integration

Let be a continuous function on real line. Then, the - 0Qrepeated integral of based at is given by single integration:

$$\int_{(a)}^{(x)} \int_{(a)}^{(x1)} \cdots\cdots \int_a^{Xn-1} f(Xn)dxn \ldots\ldots dx2dx1$$

$$= \frac{1}{(n-1)} \int_a^x (x-t)^{n-1} f(t)dt$$

Converting BVP to Fredholm Integral Equations

The method is similar to that discussed in previous section with some exceptions that are related to the boundary conditions. We demonstrate this method with an illustration.

Example1. Find the integral equation corresponding to the boundary value problem (BVP).

$$y''(x) + \lambda y(x) = 0, \qquad y(0) = 0\; y(1) = 0$$

Kernels

Types of Kernels The following special cases of the kernel of an integral equation are of main interest:-

- Symmetric Kernel
- Separable Kernel
- Resolvent Kernel
- Iterated Kernels

Symmetric Kernel

A Kernel $k(s,t)$is symmetric (or complex symmetric or Hermitian) if

$$k(s,t)= k*(t,s)$$

Where the asterisk denotes the complex conjugate. For a real kernel, this coincides with definition

$$k(s,t)= k(t,s)$$

Separable or Degenerate Kernel

A kernel $k(s,t)$is called separable or degenerate if it can be expressed as the sum of a finite number of terms, each of which is the product of a function of only and a function of only, that is

$$k(s,t) = \sum_{i=1}^{n} ai(s)bi(s)$$

Remark.

The functions $ai(s)$ can be assumed to be linearly independent, otherwise the number of terms in relation can be reduced (by linear independence it is meant that, if $c_1a_1+ c_2a_2+\ldots\ldots+ c_na_n= 0$. where c_i are arbitrary constants, then $c_1 = c_2 = \cdots = c_n = 0$).

Resolvent Kernel

Suppose solution of integral equations

$$y(x) + f(x) + \lambda \int_a^b k(x,t)\, y(t)dt$$

$$y(x) + f(x) + \lambda \int_a^x k(x,t)\, y(t)dt$$

$$y(x) + f(x) + \lambda \int_a^b R(x,t;\lambda)y(t)dt$$

$$y(x) + f(x) + \lambda \int_a^b T(x,t;\lambda)y(t)dt$$

Then $R(x,t;\lambda)$or $T(x,t;\lambda)$is called the Resolvent kernel or reciprocal kernel of the given integral equation.

Iterated Kernels

Consider Fredholm integral equation of the second kind

$$y(x) + f(x) + \lambda \int_a^b k(x,t)\, y(t)dt$$

Then, the iterated kernels:572 $kn(x,t)n$, - = 1,2,3, ⋯ are defined as follows

572$k1(x,t) = k(x,t)$

$$kn(x,t) = \int_a^b k(x,z)\, kn-1(x,t)dt \quad n = 2,3 \ldots.$$

Method and material: We conducted this research paper by observing the different types of reviews, as well as conducting and evaluating literature review papers.

RESULT

A new general integral transform that covers all classes of integral transforms in the class of Laplace transforms. We investigated the application of this new transformation to solving ODEs with constant and variable coefficients. And the fractional order can easily handle for differential equations. We have discussed the advantages and disadvantages of other integral transforms defined during the last 2 decades. We proved the related theorems for this new transformation.

FUTURE ASPECT & CONCLUSION:

In this paper, we have discussed the classifications of integral equations such as Fredholm Integral Equation, Volterra Integral Equation and many others along with the kinds of particular equation. After that we studied the relation between Integral equation and Differential equation with the conversion of Initial Value Problem into Volterra Integral Equation and Boundary Value Problem into Fredholm Integral Equation. We also discussed the types of kernels in integral equations which plays an important role in finding the solution of the given systems.

Acknowledgment:

We I would like thank the editor and reviewers for their time spent on reviewing my manuscript and their comments helping me improving the article.

Compliance with Ethics Requirements: *This article does not contain any studies with human or animal subjects.*

Funding: - Not applicable.

Financial Support and Sponsorship: Nil.

Declaration of Competing Interest: The authors declare that they have no conflict of interest.

Conflict of interest statement

Authors declare that they do not have any conflict of interest.

REFERENCES

[1] K.S. Aboodh The new integral transform aboodh transform Global J Pure Appl Mathe, 9 (1) (2013), pp. 35-43

[2] N. Abbas, M.Y. Malik, M.S. Alqarni, S. Nadeem Study of three dimensional stagnation point flow of hybrid nanofluid over an isotropic slip surface Physica A, 554 (2020), p. 124020

[3] S.A.P. Ahmadi, H. Hosseinzadeh, A.Y. Cherati A new integral transform for solving higher order linear ordinary differential equations Nonlinear Dyn Syst Theory, 19 (2) (2019), pp. 243-252

[4] U. Ali, M.Y. Malik, K.U. Rehman, M.S. Alqarni Exploration of cubic autocatalysis and thermal relaxation in a non-Newtonian flow field with MHD effects Physica A, 549 (2020), p. 124349

[5] S.A.P. Ahmadi, H. Hosseinzadeh, A.Y. Cherati A new integral transform for solving higher order linear ordinary Laguerre and Hermite differential equations Int J Appl Comput Math, 5 (2019), p. 142, 10.1007/s40819-019-0712-1

[6] M.M. Abdelrahim Mahgoub The new integral transform mohand transform Adv Theoret Appl Mathe, 12 (2) (2017), pp. 113-120

[7] M.M. Abdelrahim Mahgoub The new integral transform sawi transform Adv Theoret Appl Mathe, 14 (1) (2019), pp. 81-87

[8] D. Baleanu, G. Wu Some further results of the laplace transform for variable-order fractional difference equations Fract Calcul Appl Anal, 22 (6) (2019), pp. 1641-1654, 10.1515/fca-2019-0084

[9] D. Baleanu, K. Diethelm, E. Scalas, J.J. Trujillo Fractional calculus: models and numerical methods World Scientific Publishing Company (2012)

[10] A. Bokhari, D. Baleanu, R. Belgacem Application of Shehu transform to Atangana-Baleanu derivatives J Mathe Comput Sci, 20 (2) (2020), pp. 101-107

[11] Chand M, Hammouch Z. Unified fractional integral formulae involving generalized multiindex Bessel function. In: 4th International Conference on Computational Mathematics and Engineering Sciences (CMES-2019). CMES 2019. Advances in Intelligent Systems and Computing, vol 1111. Springer, Cham, 2020.

[12] I. Cho, K. Hwajoon The solution of Bessel's equation by using Integral Transform Appl Mathe Sci, 7 (122) (2014), pp. 6069-6075

[13] B. Davies Integral transforms and their applications Springer, New York, NY (2002)

[14] H. Eltayeb, A. Kiliman, B. Fisher A new integral transform and associated distributions Integral Transforms Special Funct, 21 (5) (2010), pp. 367-379

[15] T.M. Elzaki The new integral transform Elzaki Transform Global J Pure Appl Mathe, 7 (1) (2011), pp. 57-64

[16] T.M. Elzaki, S.M. Elzaki, E.M.A. Hilal Elzaki and sumudu transforms for solving some differential equations Global J Pure Appl Mathe, 8 (2) (2012), pp. 167-173

[17] H. Jafari, S. Sadeghi Roushan, A. Haghbin The Laplace decomposition method for solving n-th order fuzzy differential equations Ann Fuzzy Mathe Informat, 7 (4) (2014), pp. 653-660

[18] H. Kamal, A. Sedeeg The new integral transform Kamal transform Adv Theoret Appl Mathe, 11 (4) (2016), pp. 451-458

[19] H. Kim On the form and properties of an integral transform with strength in integral transforms Far East J Mathe Sci, 102 (11) (2017), pp. 2831-2844

[20] Kim H. The intrinsic structure and properties of laplace-typed integral transforms. Mathe Probl Eng, 2017, Article ID 1762729, 8 p.

[21] Z.H. Khan, W.A. Khan N-transform properties and applications NUST J Eng Sci, 1 (1) (2008), pp. 127-133

[22] M. Khan, T. Salahuddin, M.Y. Malik, M.S. Alqarni, A.M. Alqahta ni Numerical modeling and analysis of bioconvection on MHD flow due to an upper paraboloid surface of revolution Physica A, 553 (2020), p. 124231

[23] L. Kexue, P. Jigen Laplace transform and fractional differential equations Appl Mathe Lett, 24 (12) (2011), pp. 2019-2023

[24] P.D. Panasare, S.P. Chalke, A.G. Choure Application of Laplace transformation in cryptography Int J Mathe Archive, 3 (7) (2012), pp. 2470-2473

[25] S. Lin, C. Lu Laplace transform for solving some families of fractional differential equations and its applications Adv Differ Equ, 2013 (2013), p. 137, 10.1186/1687-1847-2013-137 9pages

[26] nLi C, Qian D, Chen YQ. On riemann-liouville and caputo derivatives, discrete dynamics in nature and society, volume 2011, Article ID 562494, 15 pages, doi: 10.1155/2011/562494.

[27] G.D. Medina, N.R. Ojeda, J.H. Pereira, L.G. RomeronFractional laplace transform and fractional calculusn Int Mathe Forum, 12 (20) (2017), pp. 991-1000

[28] S. Nadeem, M.Y. Malik, N. Abbas Heat transfer of three-dimensional micropolar fluid on a Riga plate Can J Phys, 98 (1) (2020), pp. 32-38

[29] I. Podlubny Fractional differential equations Academic Press, San Diego (1999)

[30] S. Rashid, Z. Hammouch, H. Kalsoom, R. Ashraf, Y.M. Chu New investigations on the generalized K-fractional integral operators Front Phys, 8 (2020), p. 25

[31] J. Singh, D. Kumar, D. Baleanu, S. Rathore An efficient numerical algorithm for the fractional Drinfeld–Sokolov–Wilson equation Appl Math Comput, 335 (2018), pp. 12-24

[32] Sweilam NH, AL-Mekhlafi SM, Baleanu D. Nonstandard finite difference method for solving complex-order fractional Burgers' equations. J Adv Res. 2020 https://doi.org/10.1016/j.jare.2020.04.007

[33] N.H. Sweilam, T.M. Al-Ajami Legendre spectral-collocation method for solving some types of fractional optimal control problems J Adv Res, 6 (3) (2015), pp. 393-403

[34] K. Shah, M. Junaid, N. Ali Extraction of laplace, sumudu, fourier and mellin transform from the natural transform J Appl Envron Biol Sci, 5 (9) (2015), pp. 1-10

[35] A. Tanveer, T. Salahuddin, M. Khan, M.Y. Malik, M.S. Alqarni Theoretical analysis of non-Newtonian blood flow in a microchannel Comput Methods Programs Biomed, 191 (2020), p. 105280

[36] G.K. Watugala Sumudu transform: a new integral transform to solve differential equations and control engineering problems Int J Math Educat Sci Technol, 24 (1) (1993), pp. 35-43

[37] X.J. Yang A new integral transform operator for solving the heat-diffusion problem Appl Mathe Lett, 64 (2017), pp. 193-197

[38] B.J. West, M. Bologna, P. Grigolini Fractional Laplace Transforms Physics of Fractal Operators. Institute for Nonlinear Science, Springer, New York, NY (2003)

[39] D. Ziane, D. Baleanu, K. Belghaba, M. Hamdi Cherif Local fractional Sumudu decomposition method for linear partial differential equations with local fractional derivative J King Saud Univ-Sci, 31 (1) (2019), pp. 1018-1364

[40] J. Zhang A Sumudu based algorithm for solving differential equations Comput Sci J Moldova, 15 (3) (2007), p. 45